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Gauge interactions in the dual standard model

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Abstract: We present a geometric argument for the transformation properties of $\mathrm{SU}(5) \rightarrow \mathrm{SU}(3) >_C \times \mathrm{SU}(2)_I \times \mathrm{U}(1)_Y/\mathbb{Z}_6)$ monopoles under the residual gauge symmetry. This strongly supports the proposal that monopoles of the dual standard model interact via a gauge theory of the standard model symmetry group, with the monopoles having the same spectrum as the standard model fermions.

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The dual standard model has been proposed as a way of unifying both matter and interaction [1]. Monopoles from the Georgi-Glashow

$$\begin{aligned}\mathrm{SU}(5) \to \mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2)) =& \\ &= \mathrm{SU}(3)_{\mathrm{C}}\times\mathrm{SU}(2)_{\mathrm{I}}\times\mathrm{U}(1)_{\mathrm{Y}}/\mathbb{Z}_6\end{aligned} \tag{1}$$

grand unification have precisely the same spectrum as the observed fermions in the standard model; it is therefore natural to associate these standard model fermions with such monopoles. In consequence of this we have calculated the gauge couplings at monopole unification [2]

$$\frac{g_{\mathrm{C}}}{g_{\mathrm{I}}} = 3\,, \qquad \frac{g_{\mathrm{Y}}}{g_{\mathrm{I}}} = \frac{2}{\sqrt{15}}\,. \tag{2}$$

Both values are satisfied by the standard model gauge couplings at a scale of a few GeV.

In this letter we examine the transformation properties of these monopoles under the residual $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ symmetry. As such we show that gauge transformations of the fundamental monopoles are entirely consistent with the fundamental representation of the standard model symmetry group. This gives strong support to the proposal that the long range interaction of these monopoles is via a gauge interaction of $\mathrm{SU}(3)_{\mathrm{C}}$, $\mathrm{SU}(2)_{\mathrm{I}}$ and $\mathrm{U}(1)_{\mathrm{Y}}$ symmetry groups.

We shall consider firstly the fundamental monopoles. These are embedded $\mathrm{SU}(2)\to\mathrm{U}(1)$ monopoles,

$$\begin{array}{ccc}\mathrm{SU}(5) & \to & \mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2)) \\ \cup & & \cup \\ \mathrm{SU}(2)_Q & \to & \mathrm{U}(1)_Q.\end{array} \tag{3}$$

It is clear that these fundamental monopoles have a degeneracy of embeddings. The purpose of this letter to quantify this degeneracy.

To quantify the space of embeddings we shall label the embedding of the fundamental monopoles. For this it will prove useful to split the su(2) algebra into components

$$\mathrm{su}(2)_Q = \mathrm{u}(1)_Q \oplus \mathcal{M}_Q\,. \tag{4}$$

Here $\mathrm{u}(1)_Q$ is the Lie algebra of $\mathrm{U}(1)_Q$, and $\mathcal{M}_Q$ is its associated orthogonal component. The direct sum is with respect to the standard inner product on su(5), given by $\langle X,Y\rangle = \mathrm{tr}XY$.

One useful label for the fundamental monopoles is their magnetic charge Q (we will see later that there is another more useful label). The magnetic charge defines the asymptotic magnetic field of a monopole,

$$B^k \sim \frac{\hat{r}^k}{r^2}Q \tag{5}$$

and is associated with the embedding

$$\mathrm{U}(1)_Q = \exp(\mathbf{R}Q) \subset \mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))\,, \tag{6}$$

normalised by

$$\exp(2\pi g_u Q) = 1\,, \tag{7}$$

with g_u the unified SU(5) gauge coupling. Additionally the embedding in eq. (6) is associated with the topology of $\mathrm{SU}(5)/\,\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$, being a non-trivial element of

$$\pi_1[\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))] = \pi_2[\mathrm{SU}(5)/\,\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))]. \tag{8}$$

Following [1], we decompose the magnetic charge into colour, weak isospin and weak hypercharge components

$$Q = \frac{1}{g_u}\left(T_{\mathrm{C}} + \frac{1}{2}T_{\mathrm{I}} + \frac{1}{3}T_{\mathrm{Y}}\right), \tag{9}$$

where $T_{\mathrm{C}} \in \mathrm{su}(3)_{\mathrm{C}}$ may be either

$$T^r_{\mathrm{C}} = i\ g\mathrm{diag}\left(+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, 0, 0\right) \tag{10}$$

$$T^g_{\mathrm{C}} = i\ \mathrm{diag}\left(-\frac{1}{3}, +\frac{2}{3}, -\frac{1}{3}, 0, 0\right) \tag{11}$$

$$T^b_{\mathrm{C}} = i\ \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, +\frac{2}{3}, 0, 0\right), \tag{12}$$

and $T_{\mathrm{I}} \in \mathrm{su}(2)_{\mathrm{I}}$ may be either

$$T^{\pm}_{\mathrm{I}} = \pm i\,\mathrm{diag}(0, 0, 0, 1, -1)\,, \tag{13}$$

whilst $T_{\mathrm{Y}} \in \mathrm{u}(1)_{\mathrm{Y}}$ may only be

$$T_{\mathrm{Y}} = i\ \mathrm{diag}\left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}\right). \tag{14}$$

The above degeneracies indicate that the fundamental monopoles form representations of $\mathrm{SU}(3)_{\mathrm{C}}$, $\mathrm{SU}(2)_{\mathrm{I}}$ and $\mathrm{U}(1)_{\mathrm{Y}}$ with the corresponding dimension. Namely the fundamental representations.

The purpose of this letter is to investigate these degeneracies. We interpret the degeneracies as being due to gauge freedom of the monopole embedding. In this light we show that the gauge degeneracy of the fundamental monopoles are consistent with the fundamental representions of the residual symmetry group S(U(3) × U(2)).

On the issue of duality, we shall show that the dual of the residual symmetry group SU(3) × SU(2) × U(1) is also consistent with the gauge degeneracy of the monopoles.

A rigid (or global) gauge transformations of the fundamental monopole is defined by an element $h \in \mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$ and transforms the magnetic field as

$$B^k \mapsto \mathrm{Ad}(h) B^k = h B^k h^{-1} \,. \tag{15}$$

Correspondingly the su(2) embedding transforms under

$$\mathrm{su}(2)_Q \mapsto \mathrm{Ad}(h)\, \mathrm{su}(2)_Q \,, \tag{16}$$

so that Q transforms appropriately. Hence the components of eq. (4) transform as

$$\begin{aligned} \mathrm{u}(1)_Q &\mapsto \mathrm{Ad}(h)\, \mathrm{u}(1)_Q \,, \\ \mathcal{M}_Q &\mapsto \mathrm{Ad}(h) \mathcal{M}_Q \,. \end{aligned} \tag{17}$$

One may see that Q is not a good quantity for examining the action of $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$ on the monopole by considering the action of elements $h \in \mathrm{U}(1)_Q$. These take $\mathrm{u}(1)_Q \mapsto \mathrm{u}(1)_Q$ identically, whilst acting non-trivially on elements of $\mathcal{M}_Q$, taking them to another element of $\mathcal{M}_Q$.

Thus to obtain all of the possible monopole embeddings we must examine the action of $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$ on $\mathcal{M}_Q$. This may be achieved by considering the action on any non-trivial element of $\mathcal{M}_Q$. Then the manifold of all equivalent fundamental monopoles under a rigid gauge transformation is

$$M(\mathcal{M}_Q) \cong \frac{\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))}{C(\mathcal{M}_Q)} \,, \tag{18}$$

with the centraliser

$$C(\mathcal{M}_Q) = \{ h \in \mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2)) : \mathrm{Ad}(h) \mathcal{M}_Q = \mathcal{M}_Q \} \tag{19}$$

representing those transformation that leave $\mathcal{M}_Q$ invariant.

We shall calculate $C(\mathcal{M}_Q)$ by considering its action on a monopole embedding. In particular consider a magnetic charge

$$Q^{r+} = \frac{1}{g_u} \left(T^r_{\mathrm{C}} + \frac{1}{2} T^+_{\mathrm{I}} + \frac{1}{3} T_{\mathrm{Y}} \right) , \tag{20}$$

having explicit components $g_u Q^{r+}_{jk} = \delta_{j1}\delta_{k1} - \delta_{j5}\delta_{k5}$. The su(2) algebra associated with this is generated by $\{g_u Q^{r+}, X^{r+}, Y^{r+}\}$, where the explicit components are

$$\begin{aligned} X^{r+}_{ij} &= \delta_{j5}\delta_{k1} - \delta_{j1}\delta_{k5} \,, \\ Y^{r+}_{ij} &= i(\delta_{j5}\delta_{k1} + \delta_{j1}\delta_{k5}) \,. \end{aligned} \tag{21}$$

Then $[X^{r+}, Y^{r+}] = 2 g_u Q^{r+}$.

To exhibit the group structure we require the generators T_{C}, T_{Y} and T_{I} expressed in a basis normalised to the topology of $\mathrm{SU}(5)/\,\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$. To this end we define

$$C=\frac{3}{2}T_{\mathrm{C}}^{r}\,,\qquad I=T_{\mathrm{I}}^{+}\,,\qquad Y=\frac{2}{5}T_{\mathrm{Y}}\,, \tag{22}$$

such that

$$\mathrm{Ad}(e^{2\pi C})\mathcal{M}_{Q}=\mathrm{Ad}(e^{2\pi I})\mathcal{M}_{Q}=\mathrm{Ad}(e^{2\pi Y})\mathcal{M}_{Q}=1\,. \tag{23}$$

In particular

$$\mathrm{Ad}(e^{\theta_{C}C})\mathrm{Ad}(e^{\theta_{I}I})\mathrm{Ad}(e^{\theta_{Y}Y})\mathcal{M}_{Q}=e^{i(\theta_{C}+\theta_{I}+\theta_{Y})}\mathcal{M}_{Q} \tag{24}$$

From this we obtain

$$C(\mathcal{M}_{Q})=\mathrm{SU}(2)_{C}\times\mathrm{U}(1)_{\mathrm{Y-I}}\times\mathrm{U}(1)_{\mathrm{I+Y-2C}}/\mathbb{Z}_{2}\,, \tag{25}$$

where $\mathbb{Z}_2$ represents an intersection between $\mathrm{SU}(2)_C$ and $\mathrm{U}(1)_{\mathrm{I+Y-2C}}$. Thus, in conclusion the manifold of rigidly gauge equivalent fundamental monopoles is

$$M(\mathcal{M}_{Q})=\frac{\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))}{\mathrm{SU}(2)_{C}\times\mathrm{U}(1)_{\mathrm{Y-I}}\times\mathrm{U}(1)_{\mathrm{I+Y-2C}}/\mathbb{Z}_{2}}\,. \tag{26}$$

This is the first main result of this letter.

We should comment that it is possible to show all of the fundamental monopoles lie within the same equivalence class. This is by associating the different monopole embeddings with the spectrum of roots corresponding to the roots of SU(5) that are not roots of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$. The action of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ upon the associated root spaces takes one monopole embedding to another. We shall discuss this fully in another publication [3].

Now we shall consider the corresponding action of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ upon a fermion in the fundamental representations of colour, weak isospin and weak hypercharge. In the standard model this corresponds to the $(u,d)_L$ quark doublet. For $f_{(u,d)_L}\in\mathbb{C}^{3\times2}$ the action is

$$f_{(u,d)_L}\mapsto h_{\mathrm{Y}}h_{\mathrm{C}}\cdot f_{(u,d)_L}\cdot h_{\mathrm{I}}\,, \tag{27}$$

with h_{Y} interpreted as a complex phase and h_{C} and h_{I} elements of $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_I$ respectively.

Consequently we may form a manifold of gauge equivalent fermion states from the actions of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ on this fermion $f_{(u,d)_L}$. The manifold is of the form

$$M(f_{(u,d)_L})\cong\frac{\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))}{C(f_{(u,d)_L})}\,, \tag{28}$$

with the stability group

$$C(f)=\{h\in\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))\ :h\cdot f=f\} \tag{29}$$

representing those transformations that leave f invariant.

Without loss of generality we shall consider acting on the specific element $f_{jk} = \delta_{j1}\delta_{k1}$. Again the generators used are normalised to the topology of S(U(3) × U(2)),

$$\begin{aligned} C &= i\,\mathrm{diag}(1,1,-2)\,, \\ I_3 &= i\,\mathrm{diag}(1,-1)\,, \\ Y &= i\,\mathrm{diag}(1,1)\,, \end{aligned} \tag{30}$$

such that $\exp(2\pi Y)f = \exp(2\pi I_3)f = \exp(2\pi C)f = 1$. In particular

$$\exp\theta_1 Y \exp\theta_3 C \cdot f \cdot \exp\theta_2 I_3 = e^{i(\theta_1+\theta_2+\theta_3)} f\,. \tag{31}$$

From this we obtain

$$C(f_{(u,d)_L}) = \mathrm{SU}(2)_C \times \mathrm{U}(1)_{\mathrm{Y-I}} \times \mathrm{U}(1)_{\mathrm{Y-2C}}/\mathbb{Z}_2 \tag{32}$$

and thus we conclude that the manifold of rigidly gauge equivalent fundamental fermions is

$$M(f_{(u,d)_L}) = \frac{\mathrm{S(U(3)\times U(2))}}{\mathrm{SU}(2)_C \times \mathrm{U}(1)_{\mathrm{Y-I}} \times \mathrm{U}(1)_{\mathrm{Y-2C}}/\mathbb{Z}_2}\,. \tag{33}$$

By comparing the above manifolds we see that both are precisely the same

$$M(\mathcal{M}_Q) = M(f_{(u,d)_L})\,. \tag{34}$$

This is our main result. It shows an equivalence between the transformation properties of fundamental monopoles and $(u,d)_L$ fermions. This supports that fundamental monopoles transform under the same representation as the $(u,d)_L$ fermion. Namely the fundamental representation of S(U(3) × U(2)).

We now consider the action of the dual group $\mathrm{S(U(3)\times U(2))}^v = \mathrm{SU(3)\times SU(2)\times U(1)}$ on the fermion f. Then the associated gauge orbit is $\mathrm{S(U(3)\times U(2))}^v/C^v(f)$. However it is clear that $C^v(f) = C(f) \times \mathbb{Z}_6$. Thus fermion gauge orbit in eq. (33) under S(U(3) × U(2)) is the same as the fermion gauge orbit under the dual group $\mathrm{S(U(3)\times U(2))}^v$. In other words the gauge orbits of monopoles are consistent with both the residual symmetry group and the dual residual symmetry group.

It is an interesting feature of the above arguments that they imply an association between the long range interactions of these monopoles and the gauge interactions of a particle transforming under the fundamental representation of $\mathrm{SU(3)_C}$, $\mathrm{SU(2)_I}$ and $\mathrm{U(1)_Y}$ gauge fields. In particular note the transformations of eq. (15) are local. Then the monopole moves around its gauge orbit under transformations of a local symmetry. This feature should be viewed as the background to our work on unification in the dual standard model [2]. There the starting assumption is that the monopoles interact via a gauge interaction, and as a consequence we derive relations between the gauge couplings at monopole unification.

Also of note is that the techniques used here relate purely to the symmetry properties of the model. Thus our derivation should be very general, and we expect that techniques used here will be applicable to other situations of interest. Other examples that should be amenable to this approach include monopoles from various symmetry breakings, and the long range interactions of vortices.

We now move on to a discussion of the gauge equivalence classes for the other monopoles. These are formed from stable bound states of fundamental monopoles [4].

Writing the magnetic charges of these other stable monopoles as

$$Q_{q_{\mathrm{Y}}} = \frac{1}{g_u}\left(q_{\mathrm{C}}T_{\mathrm{C}} + q_{\mathrm{I}}T_{\mathrm{I}} + q_{\mathrm{Y}}T_{\mathrm{Y}}\right), \tag{35}$$

where a particular state is labelled by its hypercharge, the following spectrum of stable monopoles is obtained:

	q_{C}	q_{I}	q_{Y}	d_{C}	d_{I}	d_{Y}	f
$(e^{2i\pi/3}, -1)$	1	1/2	1/3	3	2	1	$(u,d)_L$
$(e^{-2i\pi/3}, 1)$	-1	0	2/3	3	0	1	$\bar{d}_L$
$(1, -1)$	0	1/2	1	0	2	1	$(\bar{\nu}, \bar{e})_R$
$(e^{2i\pi/3}, 1)$	1	0	4/3	3	0	1	u_R
$(e^{-2i\pi/3}, -1)$	–	–	–	–	–	–	–
$(1, 1)$	0	0	2	0	0	1	$\bar{e}_L$

The degeneracies of each bound state has also been included, relating to the degeneracy in eqs. (10), (11), (12) and eq. (13). We have also included the standard model fermions that have the same charges as the monopoles.

We shall consider firstly the gauge equivalence classes of the fermions in the standard model. As before these are of the form

$$M(f) = \frac{\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))}{C(f)}, \tag{36}$$

with $C(f)$ the centraliser of the fermion's gauge transformations, namely

$$C(f) = \{h \in \mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2)) : h\cdot f = f\}. \tag{37}$$

A calculation analogous to that carried out in the first part of this letter gives:

$$\begin{aligned} C(f_{(u,d)_L}) &= \mathrm{SU}(2)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{Y-I}} \times \mathrm{U}(1)_{\mathrm{Y-2C}}/\mathbb{Z}_2 \\ C(f_{\bar{d}_L}) &= \mathrm{SU}(2)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{I}} \times \mathrm{U}(1)_{\mathrm{Y-2C}}/\mathbb{Z}_2 \\ C(f_{(\bar{\nu},\bar{e})_L}) &= \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{Y-I}} \\ C(f_{u_R}) &= \mathrm{SU}(2)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{I}} \times \mathrm{U}(1)_{\mathrm{Y-2C}}/\mathbb{Z}_2 \\ C(f_{\bar{e}_L}) &= \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{I}}\,. \end{aligned} \tag{38}$$

We shall now turn to the problem of determining the gauge equivalence classes of the monopoles. However, our analysis is complicated by the fact that higher charged stable monopoles are not embedded monopoles. This point was crucial for our analysis in the first part of this letter, where we associated a $\mathcal{M}_Q$ with the monopole embedding and described the group actions upon this.

Instead we shall deal only with the magnetic charge of the monopoles. Observe that for the $(u,d)_L$ fundamental monopole the subgroup of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ that leaves the magnetic charge invariant is

$$C(Q_{1/3}) = \{h \in \mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2)) : \mathrm{Ad}(h)Q_{1/3} = Q_{1/3}\}\,, \tag{39}$$

for which explicit calculation yields

$$C(Q_{1/3}) = \mathrm{SU}(2)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}/\mathbb{Z}_6\,. \tag{40}$$

It is clear that this is related to $C(\mathcal{M}_Q)$ by

$$C(Q_{1/3}) = C(\mathcal{M}_Q)\times\mathrm{U}(1)_Q/\mathbb{Z}_6\,. \tag{41}$$

Physically this represents $\mathrm{U}(1)_Q$ acting trivially upon Q, whilst acting non-trivially upon the monopole. Whilst $\mathrm{U}(1)_Q$ does not appear in the action of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ on the magnetic charge, it is still important in its action upon the monopole embedding.

Now we verified in the first part of this letter that $C(\mathcal{M}_Q) = C(f_{(u,d)_L})$. In fact this was all that was needed to prove that the monopole and fermion gauge equivalence classes were the same. Taking the analogy of this we shall show that

$$C(f_{(u,d)_L})\times\mathrm{U}(1)_Q/\mathbb{Z}_6 = C(Q)\,, \tag{42}$$

for each of the respective higher charge monopoles and their associated fermions.

The magnetic charges of the higher charge monopoles are given by the table above. From this we calculate

$$\begin{aligned}
Q^{r+}_{1/3} &= i\,\mathrm{diag}(1,0,0,0,-1)\\
Q^{r}_{2/3} &= i\,\mathrm{diag}(0,1,1,-1,-1)\\
Q^{+}_{1} &= i\,\mathrm{diag}(1,1,1,-1,-2)\\
Q^{r}_{4/3} &= i\,\mathrm{diag}(2,1,1,-2,-2)\\
Q_{2} &= i\,\mathrm{diag}(2,2,2,-3,-3)\,.
\end{aligned} \tag{43}$$

which yields their respective stability groups

$$\begin{aligned}
C(Q_{1/3}) &= \mathrm{U}(2)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}/\mathbb{Z}_6\\
C(Q_{2/3}) &= \mathrm{U}(2)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}/\mathbb{Z}_6\\
C(Q_{1}) &= \mathrm{SU}(3)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}/\mathbb{Z}_6\\
C(Q_{4/3}) &= \mathrm{U}(2)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}/\mathbb{Z}_6\\
C(Q_{2}) &= \mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}/\mathbb{Z}_6\,.
\end{aligned} \tag{44}$$

From this it is a simple matter to see that eq. (42) holds.

However, the above does not rigourously prove equivalence of their gauge equivalence classes; to do that one must examine the specific form of the monopoles, as in the first part of this letter. Nevertheless the verification that eq. (42) holds for each of the monopoles and their respective fermion counterparts constitutes a strong indication that the gauge equivalence classes are the same.

We conclude this letter with a last remark about the structure of the higher charge monopole equivalence classes. Consideration of the above equations reveals that the $(\nu, e)_L$ monopole does not transform under colour symmetry. Thus it is naturally associated with fundamental monopoles arising from the symmetry breaking

$$\mathrm{SU}(3) \to \mathrm{U}(2) = \mathrm{SU}(2)_{\mathrm{I}} \times \mathrm{U}(1)_{\mathrm{Y}}/\mathbb{Z}_2\,. \tag{45}$$

These monopoles are again given by embedding an SU(2) monopole. Then the gauge equivalence class of such fundamental monopoles are determined by analogous methods to those in the first part of this letter

$$M(\mathcal{M}_{Q_1}) \cong \frac{\mathrm{U}(2)}{\mathrm{U}(1)_{Y-I}}\,. \tag{46}$$

This is the same manifold as the gauge equivalence class of $(\nu, e)_L$ fermions.

Likewise the monopoles associated with u_R and d_R do not transform under isospin symmetry and it is natural to associate them with monopoles arising from

$$\mathrm{SU}(4) \to \mathrm{U}(3) = \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_Y/\mathbb{Z}_3\,. \tag{47}$$

Here the d_R monopoles is given by embedding an SU(2) monopole, whilst the u_R is interpreted as a bound state of two of these. Their gauge equivalence class is

$$M(\mathcal{M}_{Q_{2/3}}) \cong \frac{\mathrm{U}(3)}{\mathrm{SU}(2)_C \times \mathrm{U}(1)_{Y-2C}/\mathbb{Z}_2}\,, \tag{48}$$

the same as for the u_R fermion.

Finally the monopole associated with e_R is associated with monopoles arising from

$$\mathrm{SU}(2) \to \mathrm{U}(1)_Y\,. \tag{49}$$

Again, trivially, this is an embedded monopole. This time the gauge equivalence class is

$$M(\mathcal{M}_{Q_2}) \cong \mathrm{U}(1)\,, \tag{50}$$

the same as for e_R.

Thus we remark that the higher charge monopoles are associated with fundamental monopoles in other symmetry breakings. Furthermore their gauge equivalence classes are calculable by similar methods to the first part of this letter. Such calculations yield the same equivalence classes as the corresponding fermions.

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